



CSE330 Section 20, Spring 2026

Quiz 5, 30 April 2026

Time: 20 Minutes

/15

Name:

ID:

MCQs (5 marks). Tick/circle the best answer.

- A linear system $Ux = b$ where U is an upper triangular matrix is best solved using:
 - Gaussian elimination
 - Forward substitution
 - ✓ Back substitution
 - LU decomposition
- What is the main purpose of pivoting in Gaussian elimination?
 - To reduce the number of arithmetic operations
 - ✓ To improve numerical stability
 - To ensure the matrix is symmetric
 - To make the matrix triangular
- In the QR decomposition $A = QR$, what property does Q satisfy?
 - Q is triangular
 - ✓ $Q^T Q = I$
 - Q is diagonal
 - Q is singular
- During Gaussian elimination, if a zero pivot is encountered, what should be done?
 - Stop the algorithm
 - ✓ Swap with a lower row having a nonzero entry
 - Replace the row with zeros
 - Multiply the row by zero
- In QR decomposition used for least squares, solving $Ax = b$ reduces to solving:
 - $Qx = b$
 - ✓ $Rx = Q^T b$
 - $Ax = Qb$
 - $R^T x = b$

Question 1: (2+6+2 Marks) Suppose you have 3 points $(1, 1), (2, 2), (3, 2)$. You want to consider the best-fit line through these points $y = mx + c$, (where c and m are unknown).

- Transform this problem into an overdeterministic linear system $Ax = b$. (Hint: here the unknown vector x is $\begin{bmatrix} c \\ m \end{bmatrix}$.)
- Compute the QR decomposition $A = QR$, where Q has orthonormal columns and R is upper triangular.
- Use the QR decomposition to find the best-fit line.

Solution. (a) The system of linear equations is

$$c + m = 1$$

$$c + 2m = 2$$

$$c + 3m = 2$$

Hence, the overdeterministic system is

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} c \\ m \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}.$$

(b) Here, $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, and $\|\mathbf{a}_1\| = \sqrt{3}$. Hence, $\mathbf{q}_1 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Now,

$$\mathbf{u}_2 = \mathbf{a}_2 - (\mathbf{a}_2 \cdot \mathbf{q}_1)\mathbf{q}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}^T \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} \right) \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - 2\sqrt{3} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

So $\mathbf{q}_2 = \frac{1}{\|\mathbf{u}_2\|} \mathbf{u}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$. Hence,

$$Q = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}.$$

$$R = \begin{bmatrix} \mathbf{a}_1 \cdot \mathbf{q}_1 & \mathbf{a}_2 \cdot \mathbf{q}_1 \\ 0 & \mathbf{a}_2 \cdot \mathbf{q}_2 \end{bmatrix} = \begin{bmatrix} \sqrt{3} & 2\sqrt{3} \\ 0 & \sqrt{2} \end{bmatrix}$$

(c) We need to solve $Rx = Q^T b$.

$$Q^T b = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

Solving $Rx = Q^T b$, we get

$$\begin{bmatrix} \sqrt{3} & 2\sqrt{3} \\ 0 & \sqrt{2} \end{bmatrix} \begin{bmatrix} c \\ m \end{bmatrix} = \begin{bmatrix} \frac{5}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \implies m = \frac{1}{2} \text{ and } c = \frac{\frac{5}{\sqrt{3}} - 2\sqrt{3} \cdot \frac{1}{2}}{\sqrt{3}} = \frac{2}{3}.$$

Therefore, the best-fit line is $y = \frac{1}{2}x + \frac{2}{3}$. ■