



Name:

ID:

MCQs (5 marks). Tick/circle the best answer.

- Hermite interpolation differs from standard polynomial interpolation because it:
 - Uses fewer nodes
 - ✓ Matches both function values and derivatives
 - Avoids polynomials entirely
 - Requires equally spaced nodes
- If we interpolate $n + 1$ distinct nodes and also match the first derivative at each node, what is the maximum degree of the Hermite interpolating polynomial?
 - n
 - $n + 1$
 - ✓ $2n + 1$
 - $2n$
- Which of the following is true about the uniqueness of the Hermite interpolating polynomial?
 - It may not be unique
 - ✓ It is unique if nodes are distinct
 - It is unique only for linear functions
 - It depends on the method used
- Which finite difference formula generally gives a more accurate approximation of $f'(x)$?
 - Forward difference
 - Backward difference
 - ✓ Central difference
 - All equally accurate
- What is the order of truncation error for the forward difference $\frac{f(x+h)-f(x)}{h}$?
 - $O(1)$
 - ✓ $O(h)$
 - $O(h^2)$
 - $O(h^3)$

Question 1: Suppose you have the function $f(x) = e^{1+x}$. You want to calculate its **second derivative** $f''(x)$ numerically at $x = 0$. You are given the following formula for “second derivative central difference”

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$$

- Using this formula, find $f''(x)$ at $x = 0$ using $h = 0.1$.
- Find the correct value of $f''(x)$ at $x = 0$, and calculate the error.
- Suppose I told you that the upper bound of error is $\frac{h^2}{12} \max |f^{(4)}|$. Calculate the upper bound for $h = 0.1$, and verify that it is indeed larger than the error you found in (b).

Solution. (a) Plugging $x = 0$ and $h = 0.1$, we have

$$\frac{f(x+h) - 2f(x) + f(x-h)}{h^2} = \frac{e^{1.1} - 2e^1 + e^{0.9}}{0.01} = 2.7205478$$

- $f(x) = e^{1+x}$, so $f'(x) = e^{1+x}$, and $f''(x) = e^{1+x}$. Therefore, the correct value of $f''(x)$ at $x = 0$ is $f''(0) = e = 2.7182818$. Therefore,

$$\text{error} = |2.7205478 - 2.7182818| = 0.0022660$$

- We have seen that $f = f'$, and hence, $f^{(4)}(x) = e^{1+x}$. Hence, the upper bound of error is

$$\frac{h^2}{12} \max |f^{(4)}| = \frac{0.01}{12} \max_{x \in [-0.1, +0.1]} |e^{1+x}| = \frac{0.01}{12} e^{1.1},$$

since e^{1+x} is increasing, and hence, the maximum is achieved at the right end of the interval. The value is

$$\frac{0.01}{12} e^{1.1} = 0.0025035,$$

which is clearly larger than the error we got in (b). ■