

CSE330 Section 20, Assignment 2

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1 Instructions

1. The deadline is **28 March 2026, during class**. There will be **no deadline extension**. Hence, don't wait for the last minute to submit.
2. Please start the assignment as soon as possible. If you start it just a day before the deadline, you might not be able to complete it.
3. If you take help from your friend (even if your friend is from a different section), or if you use ChatGPT to help you understand the material, you have to mention it on top of the first page of your assignment. Failing to do so will be considered as plagiarism and will have consequences as per the plagiarism policy.
4. Copying AI-generated answers without understanding or attribution will be treated as academic misconduct.
5. The total mark of this assignment is 15. However, there is 1 bonus question that is worth 2 points. Attempt the bonus questions only after you have solved all the other problems. If you get more than 15, then the extra points will be adjusted with other assignments.
6. Please feel free to reach out to me if you have any questions.

2 Problems

Problem 1 ($1 + 2 + 4 + 4 + 1 = 12$ marks). Consider $f(x) = x \ln(1 + x)$.

- (a) Write the full Taylor series of $f(x)$ near $x = 0$. You may look up the Taylor series for $\ln(1 + x)$ near $x = 0$.
- (b) Write the degree-3 Taylor polynomial $p_3(x)$ near $x = 0$. (Hint: $p_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{3!}x^3$.)
- (c) Find the degree-3 interpolating polynomial $p_3(x)$ with nodes

$$x_0 = 0, \quad x_1 = 0.25, \quad x_2 = 0.5, \quad x_3 = -0.5.$$

You may use either Vandermonde method, or Lagrange method, or Newton's divided difference method. (However, one of these methods will make your task easier for the next problem. Which one is it?)

- (d) Find the degree-3 Hermite polynomial $p_3(x)$ with nodes $x_0 = 0, x_1 = 0.5$. (You may need the derivative $f'(x) = \frac{x}{1+x} + \ln(1+x)$.)
- (e) Which one of the polynomials in (b), (c), (d), (e) gives the least **relative error** while calculating $f(0.2)$?

Problem 2 (3 points). We have seen that the error for finding Central-difference is (keeping rounding error in mind) in the order of

$$\frac{h^2}{6} + \frac{\varepsilon_M}{h}. \quad (2.1)$$

Given that machine epsilon is $\varepsilon_M = 1.11 \times 10^{-16}$, find the h that minimizes the error. Also find the minimized error.

(Hint: how do you calculate the maximum/minimum of a function? Recall from MAT110 that if x_* is the point at which $f(x)$ is minimum, then we must have $f'(x_*) = 0$ and $f''(x_*) > 0$.)

Problem 3 (Bonus, 2 points). We have seen in class that interpolation error is

$$f(x) - p_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!} (x - x_0)(x - x_1) \cdots (x - x_n), \quad (2.2)$$

for some c . Suppose we are adding one more node $(x_{n+1}, f(x_{n+1}))$. Can you relate the interpolation error with the divided-difference $f[x_0, x_1, \dots, x_{n+1}]$?