

CSE330, 12 March

Nonlinear equation solving.
(Not in midterm).

linear eqn

$$7x + 3 = 0$$
$$\Rightarrow x = -\frac{3}{7}$$

$$x^2 - 5x + 6 = 0$$

$$\Rightarrow 9x^2 + 17x + 73 = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic eqn

$$ax^2 + bx + c = 0$$

$$x^3 - 7x + 6 = 0$$

$$x^5 + 9x^3 + 3x + 1 = 0$$

Quiz 3 rescheduled

28 March;

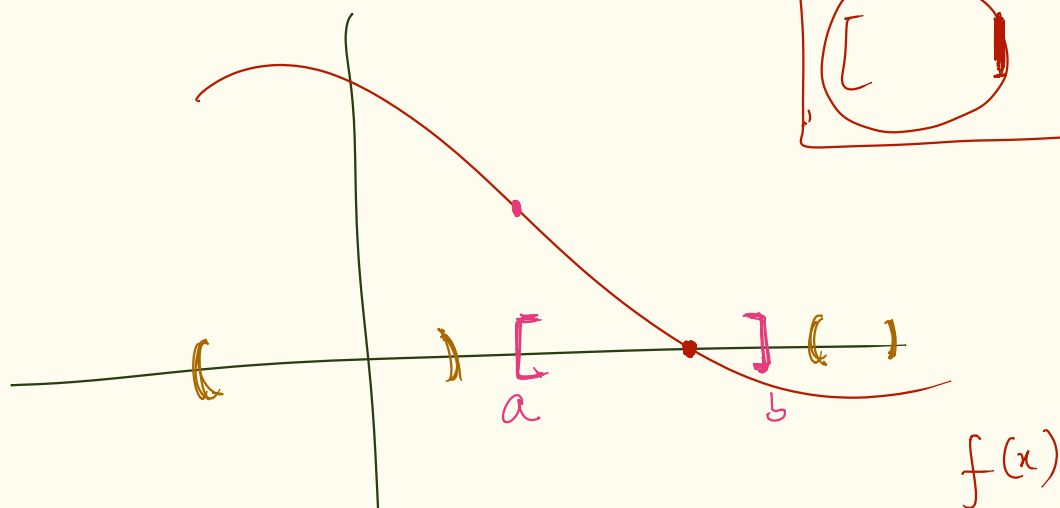
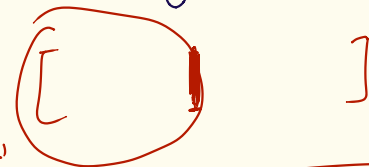
Hermite interpolation,
Diff.

Ass 2: Deadline 28M.

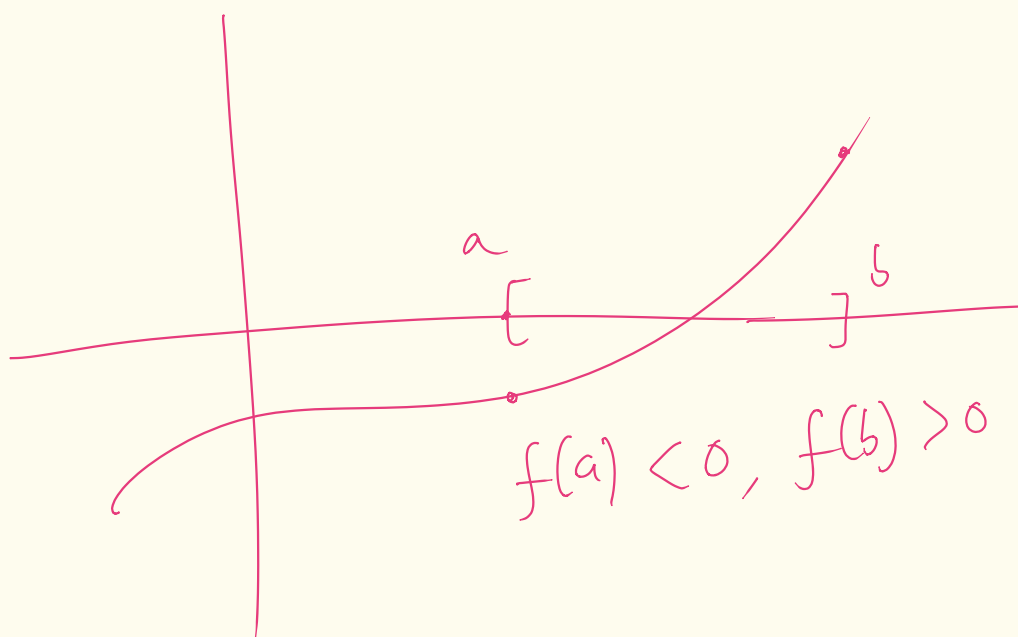
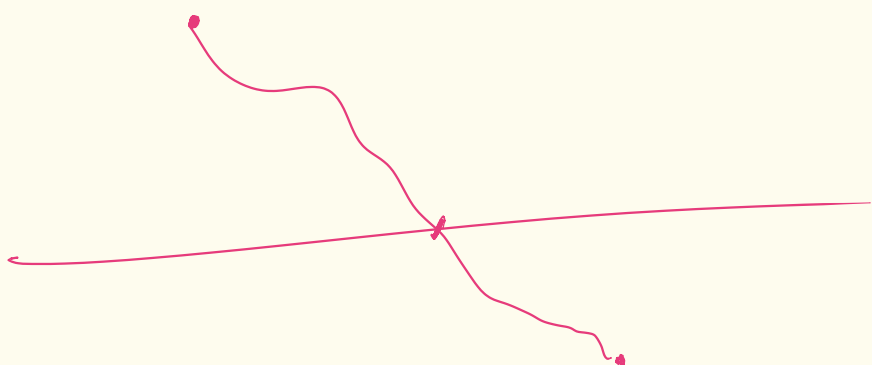
Continuous function

$$f(x) = 0$$

(Binary search)



$f(a) > 0$
 $f(b) < 0$ } therefore, there is
a root in $[a, b]$.

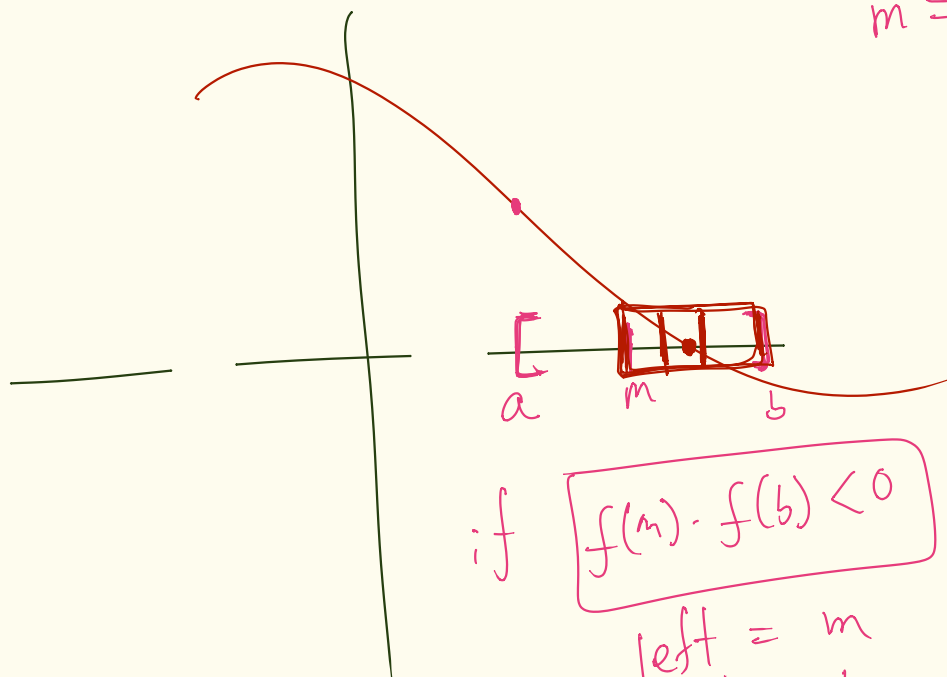


$$f(a) < 0, f(b) > 0$$

* $f(a), f(b)$ have different sign.

$$\boxed{f(a)f(b) < 0}$$

$$m = \frac{a+b}{2}$$



if $\boxed{f(m) \cdot f(b) < 0}$:

left = m
right = b

$f(m) = 0 \Rightarrow \text{root} = m$ ✓

↑ (Found exact Root)

a b
infinitely many real numbers



Diff = $0.000001 = 1 \times 10^{-6}$

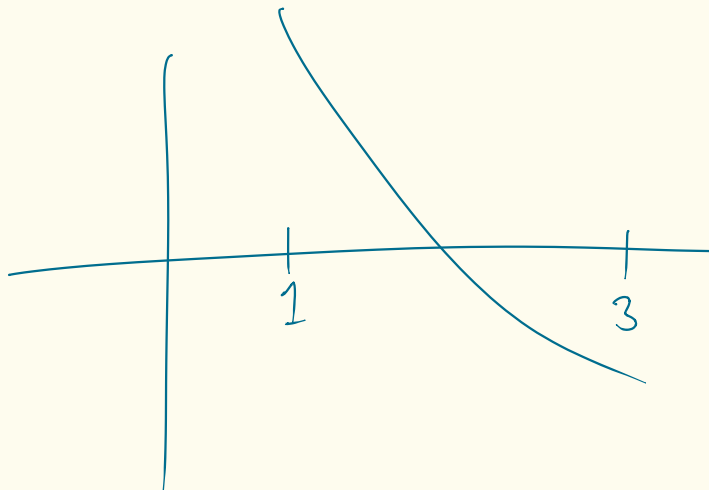
When $|b-a|$ is very small, say δ ,
 we declare $\left(\frac{a+b}{2}\right)$ to be the root.
 we are accurate upto δ .

Example 1: $f(x) = \frac{1}{x} - 0.5$.

($x=2$ actual root).

$$a = 1; \quad f(a) = 0.5 > 0$$

$$b = 3; \quad f(b) < 0.$$



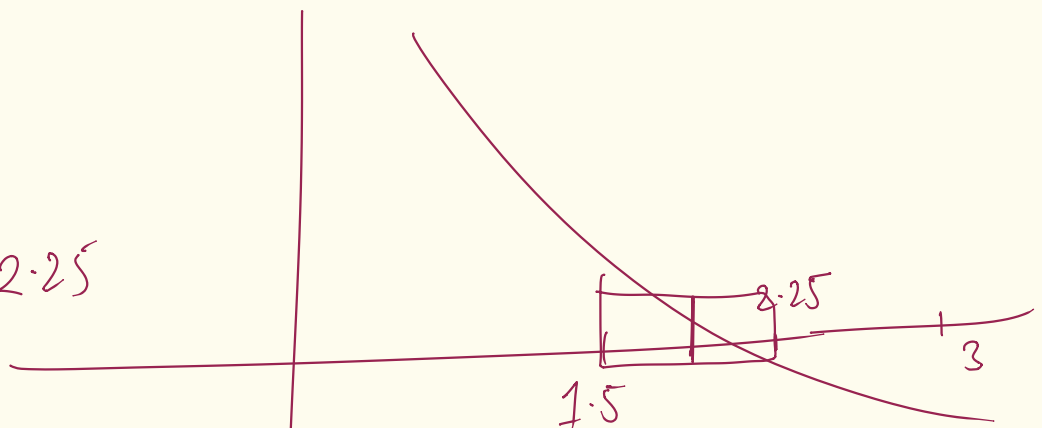
$$m = \frac{a+b}{2} = 2.$$

$$f(2) = 0. \quad \checkmark$$

$$a = 1.5$$

$$b = 3$$

$$m = \frac{1.5+3}{2} = 2.25$$



ନୂଆ $a = 1.5$

ନୂଆ $b = 2.25$

$$m = \frac{1.5+2.25}{2} = 1.875$$

$$[1.875, 2.25] \rightarrow m = \frac{1.875 + 2.25}{2} = 2.0625$$

$$f(1.875) \cdot f(2.0625) < 0$$

$$\Rightarrow [1.875, 2.0625] \text{ half}$$

$$\Rightarrow \dots \dots \dots \text{diff} = 0.0011 = 1.1 \times 10^{-3}$$

$$[1.999, 2.0001]$$

Since the difference is "very small",
(error tolerance)

$$\text{we declare root} = \frac{2.0001 + 1.999}{2}$$

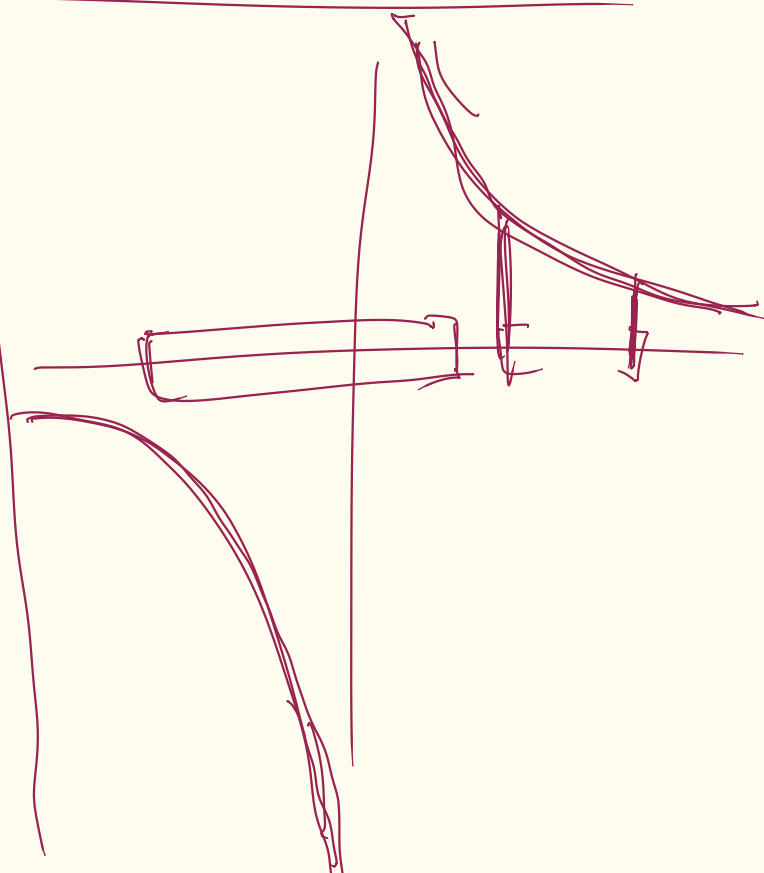
Example: $f(x) = \frac{1}{x}$;

interval $[-1, 1]$:

$$f(a) < 0$$

$$f(b) > 0$$

$\Rightarrow f(x)$ is NOT continuous in $[-1, 1]$.

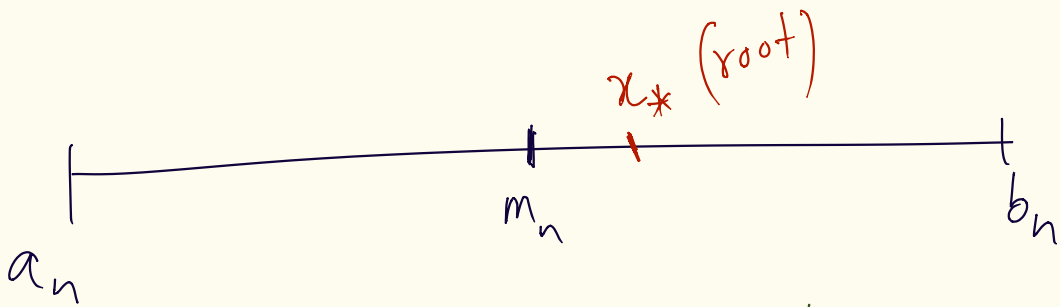


If error tolerance δ is given,
can we find out how many iteration is needed?

Suppose we're doing n iterations.

$$[a_0, b_0] \longrightarrow [a_n, b_n]$$

$$|b_n - a_n| = \frac{|b_0 - a_0|}{2^n}$$



$$\text{error} = |m_n - x_*| \leq \frac{|b_n - a_n|}{2}$$

maximized

when $x_* = a_n$

or $x_* = b_n$

$$\text{error} \leq \frac{|b_n - a_n|}{2} = \frac{|b_0 - a_0|}{2^{n+1}}$$

Suppose error tolerance $= 10^{-6}$
(error has to be less than 10^{-6}).

$$a_0 = 1.5, \quad b_0 = 3.$$

$\Rightarrow$ find n for which

$$\frac{|b_0 - a_0|}{2^{n+1}} \leq 10^{-6}$$

$$\Rightarrow 2^{n+1} \geq \frac{|b_0 - a_0|}{10^{-6}} = 1.5 \times 10^6$$

$$\Rightarrow n+1 \geq \log_2 (1.5 \times 10^6)$$

$$\Rightarrow n \geq \log_2 (1.5 \times 10^6) - 1 \\ = 19.5$$

$$n = 20.$$

$$\delta = 1.1 \times 10^{-16},$$

$$b_0 = 3, \quad a_0 = 1.5.$$

How many iteration would it take?

(53)

